Differential Geometry

♠ 2014/09/28

1, Suppose r is an arc length parametrized curve with the property that

$$|r(s)| \leq |r(s_0)| = R$$

for all s sufficiently close to s_0. Prove that $k(s_0) \geq \frac{1}{R}$. (Hint: Consider the function $f(s) = |r(s)|^2$.)

♠ 2014/10/23

2, (Hyperbolic Half Plane) Let

$$\mathfrak{H} = \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$$

be the upper half plane equipped with the first fundamental form

$$I = \frac{\mathrm{d}x^2 + \mathrm{d}y^2}{y^2}.$$

(1), Compute all Christoffel symbols Γ_{ij}^k.

(2)*, Show that the half-lines parallel to the imaginary axis and the semicircles with centres on the real axis are geodesics.

(3), Compute the Gaussian curvature K.

♠ 2014/11/11

3, Compute the Gaussian curvatures K with the following the first fundamental forms (Note that c is a constant):

(1),

$$I = \frac{du^2 + dv^2}{\left\{1 + \frac{c}{4}(u^2 + v^2)\right\}^2}.$$

(2),

$$I = du^2 + e^{\frac{2u}{c}} dv^2.$$

♠ 2014/11/20

4, Let $M \subset \mathbb{R}^3$ be a compact non-empty regular surface.

(1), Let $Q \in \mathbb{R}^3 - M$. Let $P \in M$ be a point with minimal distance from Q. Then $(Q - P)$ is perpendicular to $T_P M$.

(2), Show that the Gauss map $\nu : M \to S^2$ is onto.

(3), Show that if the Gauss map is injective, then $K \geq 0$.

(4), Improve (1) and show that the Gauss map restricted to

$$M^+ := \{x \in M \mid K(x) \geq 0\}$$

is onto. (5),

$$\int_{M^+} K dA \geq 4\pi.$$

♠ 2014/11/27

5, Let $M \subset \mathbb{R}^3$ be a regular surface, $\nu : M \to \mathbb{R}^3$ a unit normal field. Show that ν is continuous if and only if ν is smooth.

6*, A regular surface M is orientable if and only if M can be covered by local parametrisations such that for all parameter transformations $\det(J(f)) > 0$. (Hint: see de Carmo's or Andrew Pressley's book)

7*, The Mobius band is not orientable. (Hint: see de Carmo's or Andrew Pressley's book)

♠ 2014/12/2

8, Using two methods compute the area ♠ in the Hyperbolic Half Plane.

Guanghua Ji · 纪广华
School of Mathematics
Shandong University
Jinan, Shandong 250100
www.prime.sdu.edu.cn/ghji/guanghuaji.htm