International Conference on Number Theory and Representation Theory

Shandong University, Weihai, China

August 2-8, 2009

http://www.math.uiowa.edu/~yey/weihai09.html

Invited Speakers:

Andrew Booker, University of Bristol
Daniel Bump, Stanford University
Brian Conrey, American Institute of Mathematics
Dorian Goldfeld, Columbia University
Muthu Krishnamurthy, The University of Iowa
Philip Kutzko, The University of Iowa
Yuk-Kam Lau, The University of Hong Kong
Lawrence Morris, Clark University
János Pintz , Alfréd Rényi Institute of Mathematics
Kai-Man Tsang, The University of Hong Kong
Jie Wu, Université de Nancy 1

Speakers:

Yonggao Chen, Nanjing Normal University
Guangshi Lü, Shandong University
Xiumin Ren, Shandong University
Zhiwei Sun, Nanjing University

Student Speakers:

Tim Gillespie, The University of Iowa
Qingfeng Sun, Shandong University

Participants:

Ce Bian, University of Bristol
Tianxin Cai, Zhejiang University
Chaohua Jia, Chinese Academy of Sciences
Hongze Li, Shanghai Jiao Tong University
Qian Lin, Massachusetts Institute of Technology
Jing Ma, Jilin University
Kejian Xu, Qingdao University
Wenguang Zhai, Shandong Normal University

Deyu Zhang, Shandong Normal University

Organizers:

Yangbo Ye, The University of Iowa and Shandong University
Jianya Liu, Shandong University

Academic Programs:

The conference will focus on analytic number theory and group representation theory. In addition to the invited lectures there will be contributed talks. Talks will be on August 3 through August 7.

Registration:

If you are planning to attend this Conference, please email the organizers. There is no registration fee. Please tell us if you are interested in giving a contributed talk.

Scientific Program

Aug 2, 2009 (Sunday)

Registration

Aug 3, 2009 (Monday)

Morning Session **Chair:** Jianya Liu

Time: 09:00-09:10

Yangbo Ye: Opening ceremony

Time: 09:10-10:00

Speaker: Daniel Bump (Stanford University)

Title: A six vertex model for Schur functions and metaplectic Whittaker coefficients

Abstract: In statistical physics the six-vertex model for 2-dimensional ice assigns a Boltzmann weight to each vertex in a plane lattice. The weight depends on six allowable orientations of spins that are assigned to the edges adjacent to the vertex. The weight of the configuration is the product of these weights over all vertices, and the partition function is the sum over all states of the system. The six-vertex model was solved by Lieb in 1967, who evaluated the partition function under the assumption that the weights are symmetric, that is, invariant under the reversal of all spins. Baxter gave a striking method of solving the model based on the so-called Yang-Baxter equation as a method of demonstrating the commutativity of certain "transfer matrices". We will show that it is possible to assign

boundary conditions and Boltzmann weights that are not symmetric, hence outside the existing theory, such that the partition function is an arbitrary Schur polynomial in given spectral (Langlands) parameters. By means of the Casselman-Shalika formula, this may be interpreted as a value of a p-adic Whittaker function. This generalizes to give a statistical interpretation of p-adic Whittaker functions on the metaplectic covers of $\mathrm{GL}(r)$. In previous work, these were interpreted as sums over crystal graphs. A key result involved the equivalence of two such definitions, roughly related by the Schützenberger involution of the crystal graph. We will show that in the statistical interpretation this key result is reinterpreted as a statement that two transfer matrices commute, and that Baxter's method based on the Yang-Baxter equation gives a new proof. This is joint work with Brubaker and Friedberg.

Time: 10:20-10:50

Speaker: Zhiwei Sun (Nanjing University)

Title: Polygonal numbers, primes and ternary quadratic forms

Abstract: For $m = 3, 4, \ldots$ the m-gonal numbers (or polygonal numbers of order m) are given by $p_m(n) = (m-2)n(n-1)/2 + n$ $(n = 0, 1, 2, \ldots)$. A famous assertion of Fermat states that any nonnegative integer is the sum of m polygonal numbers of order m. Recently the speaker conjectured that any nonnegative integer can be written as the sum of two squares and a pentagonal number, as the sum of a triangular number, an even square and a pentagonal number, and as the sum of a square, a pentagonal number and a hexagonal number. In this talk we give a survey of the speaker's various conjectures and results involving polygonal numbers, primes and ternary quadratic forms. For example, the speaker conjectured that each integer $n > 1$ relatively prime to 6 has the form $p + 6x^2$ with p a prime, and the speaker proved that any positive integer congruent to 1 (mod 6) has the form $x^2 + 3y^2 + 24z^2$ where x, y, z are integers.

Time: 11:10-12:00

Speaker: Philip Kutzko (The University of Iowa)

Title: The representation theory of p-adic groups: Background and survey of results

<u>Afternoon Session</u> **Chair:** Philip Kutzko

Time: 14:00-14:50

Speaker: Kai-Man Tsang (The University of Hong Kong)

Title: An extension of the Brun-Titchmarsh Inequality

Abstract: The celebrated Brun-Titchmarsh inequality

$$\pi(x; k, a) = \sum_{\substack{p \le x \\ p \equiv a \pmod{k}}} 1 \ll \frac{x}{\varphi(k) \log \frac{x}{k}}$$

which holds uniformly in $k < x$ is a useful supplement to the Siegel-Walfisz theorem, and the Bombieri-Vinogradov theorem. In this talk, we shall

present a joint work with T.H. Chan and S.K.K. Choi on an extension of the Brun-Titchmarsh inequality to numbers with given number of prime factors.

Time: 15:10-16:00

Speaker: Andrew Booker (University of Bristol)

Title: An introduction to computational aspects of automorphic forms

Abstract: I will give an introduction to the computational theory of automorphic forms and describe some recent advances as well as problems for the future.

Aug 4, 2009 (Tuesday)

Morning Session **Chair:** Daniel Bump

Time: 09:00-09:50

Speaker: Philip Kutzko (The University of Iowa)

Title: The representation theory of p-adic groups: Plancherel Theorem with applications to local number theory

Time: 10:10-10:40

Speaker: Xiumin Ren (Shandong University)

Title: Exponential sums concerning primes and the Möbius function

Abstract: Exponential sums appear naturally in number theory. Nontrivial estimates of such sums lead to various equi-distribution theorems, as well as solutions to problems in additive number theory. In this talk, by using analytic method, we give estimates for exponential sums concerning primes and the Möbius function. Applications of these estimates in number theory will be illustrated.

Time: 11:00-11:50

Speaker: Muthu Krishnamurthy (The University of Iowa)

Title: A strengthening of the GL(2) converse theorem

Abstract: I will present a version of the Jacquet-Langlands converse theorem with relaxed conditions on the twists by ramified idele class characters. This is joint work with Andrew Booker.

Afternoon Session **Chair:** Kai-Man Tsang

Time: 14:00-14:50

Speaker: János Pintz (Alfréd Rényi Institute of Mathematics)

Title: The parity problem and some conjectures of Erdös on consecutive integers

Abstract: Erdös had many favorite problems on consecutive integers We will discuss among others the celebrated Erdös-Mirsky conjecture on consecutive values of the divisor function, stating $d(x) = d(x+1)$ for infinitely many integers x and its generalization for arbitrary integer shifts n, that

is, whether for a fixed n the equation $d(x) = d(x+n)$ holds for infinitely many values of x. Although (following the first breakthrough by C. Spiro) these problems were solved later by Heath-Brown (for $n = 1$) and G. Pinner (for all values of n), many problems remained open, for example, the analogue of Pinner's result when the number of divisors is substituted by the number of different prime divisors (the analogous problem for $n = 1$ was recently solved by J.-C. Puchta and it was generalized for many other values of n by Y. Buttkewitz). Earlier methods did not allow generalizations when the number of divisors or prime divisors is prescribed, due to the parity barrier. It will be presented in the lecture, how (similarly to small gaps between primes) Selberg's sieve can lead to a unified method to solve these problems. This is a joint work with S.W. Graham, D. Goldston and C. Yildirim.

Time: 15:10-15:30

Speaker: Tim Gillespie (The University of Iowa)

Title: A prime number theorem for Rankin-Selberg L-functions over number fields

Abstract: We deduce a prime number theorem for the Rankin-Selberg L-function attached to two automorphic cuspidal representations π of $\mathrm{GL}_n/\mathbb{E}$ and π' of $\mathrm{GL}_m/\mathbb{E}$ where $\mathbb{E}$ is a finite Galois extension of $\mathbb{Q}$. Using a result of Arthur and Clozel we do a similar calculation for two automorphic cuspidal representations π of $\mathrm{GL}_n/\mathbb{E}$ and π' of $\mathrm{GL}_m/\mathbb{F}$ where $\mathbb{E}$ and $\mathbb{F}$ are cyclic Galois extensions and π and π' are invariant under the Galois action.

Time: 15:40-16:00

Speaker: Qingfeng Sun (Shandong University)

Title: On Cusp Form Coefficients in Nonlinear Exponential Sums

Abstract: Let f be either a holomorphic Hecke eigenform of weight κ for $SL_2(\mathbb{Z})$ with

$$f(z) = \sum_{n=1}^{\infty} \lambda(n) n^{\frac{\kappa-1}{2}} e(nz),$$

or a Maass Hecke eigenform for $SL_2(\mathbb{Z})$ with Laplace eigenvalue $\frac{1}{4} + \nu^2$. In the latter case,

$$f(z) = 2\sqrt{y} \sum_{n \neq 0} \rho(n) K_{i\nu}(2\pi|n|y) e(nx).$$

Here $K_{i\nu}$ is the modified Bessel function of the third kind and $e(z) = e^{2\pi i z}$. In this paper we study the cancelation of the coefficients $\lambda(n)$ or $\rho(n)$ in nonlinear exponential sums with amplitude n^θ, $0 < \theta \leq \frac{1}{2}$.

Aug 5, 2009 (Wednesday)

Morning Session **Chair:** János Pintz

Time: 09:00-09:50

Speaker: Dorian Goldfeld (Columbia University)

Title: Fourier expansions of newforms on GL(2, $\mathbb{Q}$) at various cusps

Abstract: We shall discuss the Fourier expansion of Hecke-Maass eigenforms for $\mathrm{GL}(2,\mathbb{Q})$ of arbitrary weight, level, and character at various cusps. It will be shown that the Fourier coefficients at a cusp satisfy certain very explicit multiplicativity relations. As an application, it is proved that a local representation of $\mathrm{GL}(2,\mathbb{Q}_p)$ which is isomorphic to a local factor of a global cuspidal automorphic representation generated by the adelic lift of a newform of arbitrary weight, level N, and character (mod N) is supercuspidal if and only if p^2 divides N and at every cusp (of width m and cusp parameter $= 0$) the mp^k Fourier coefficient, at that cusp, vanishes for all sufficiently large positive integers k. This is joint work with Joe Hundley and Min Lee.

Time: 10:10-10:40

Speaker: Guangshi Lü (Shandong University)

Title: The Average Order of Some Number-Theoretic Functions

Abstract: Let $N_f(n)$ be the number of solutions of the congruence $f(x) \equiv 0 \,(\mathrm{mod}\ n)$, where $f(x)$ is an irreducible polynomial with integral coefficients. Let $Z(s)$ be the Epstein zeta function of a positive definite quadratic form, $a_k(n)$ be the nth coefficient of the kth power of $Z(s)$. In this talk I shall introduce some recent progress on sums $\sum_{n\le x} N_f(n)$ and $\sum_{n\le x} a_k(n)$.

Time: 11:00-11:50

Speaker: Brian Conrey (American Institute of Mathematics)

Title: 42

Abstract: In recent joint work with Iwaniec and Soundararajan we use what we call the 'asymptotic large sieve' to evaluate the sixth moment of Dirichlet L-functions, suitably averaged on the critical line. The main term involves the number 42 - a number which played a key motivating role in work over the last 10 years aimed at developing the links between random matrix theory and analytic number theory.

Afternoon Session **Chair:** Dorian Goldfeld

Time: 14:00-14:30

Speaker: Yonggao Chen (Nanjing Normal University)

Title: Representation Functions and Partitions of Natural Numbers

Abstract: For a set A of nonnegative integers the representation functions $R_2(A,n)$, $R_3(A,n)$ are defined as the number of solutions of the equation $n = a + a'$, $a, a' \in A$ with $a < a'$, $a \le a'$, respectively. We shall talk some recent results on $R_2(A,n)$, $R_3(A,n)$.

Time: 15:00-16:50

Speaker: Jie Wu (Université de Nancy 1)

Title: On modular signs

Abstract: We consider two questions related to the signs of Hecke eigenvalues of classical modular forms. One problem is to determine to what

extent those signs, for suitable sets of primes, determine uniquely the modular form, and we give both individual and statistical results. The second problem, which has been considered by a number of authors, is to determine the size, in terms of the conductor and weight, of the first sign-change of Hecke eigenvalues. We also give individual and statistical results. In particular we improve the recent estimate of Iwaniec, Kohnen & Sengupta.

Aug 6, 2009 (Thursday)

Morning Session **Chair:** Brian Conrey

Time: 09:00-09:50

Speaker: Lawrence Morris (Clark University)

Title: Level zero Hecke algebras and parabolic induction

Abstract: I shall discuss some published work of P.C. Kutzko and myself. In this work we use the theory of types and covers to give a proof of a more precise version of a result of Shahidi concerning the reducibility of a parabolically induced representation in the case that the inducing representation is a level zero supercuspidal representation of GL(N). The proof is entirely local and independent of characteristic (excluding residual characteristic 2) and illustrates a technique for determining the reducibility of parabolically induced representations which should be applicable to a wide variety of situations.

Time: 10:10-11:00

Speaker: Yuk-Kam Lau (The University of Hong Kong)

Title: Sum of Fourier coefficients of Cusp Forms

Abstract: Let $t_\varphi(n)$ denote the n-th normalized Fourier coefficient of a primitive holomorphic or Maass cusp form φ for the full modular group. Our interest is to understand its value in mean sense. Assuming the validity of suitable conjectures, we evaluate the high moments $\sum_{n\le x} t_\varphi(n)^j$ and the mean value $\sum_{n\le x} t_\varphi(n^j)$ over j-th powers.

Aug 7, 2009 (Friday)

Sightseeing

Aug 8, 2009 (Saturday)

Departure