## EXERCISE 2

1. Let $A B C$ be a hyperbolic triangle with side lengths $a=|B C|, b=|A C|, c=|A B|$ and angles $\alpha=\angle C A B, \beta=\angle A B C, \gamma=\angle A C B$. Show that

$$
\begin{gathered}
\frac{\sin \alpha}{\sinh a}=\frac{\sin \beta}{\sinh b}=\frac{\sin \gamma}{\sinh c}, \\
\sin \alpha \sin \beta \cosh c=\cos \alpha \cos \beta+\cos \gamma
\end{gathered}
$$

and

$$
\text { area } A B C=\pi-\alpha-\beta-\gamma
$$

2. Show that the hyperbolic area of a disc of radius $r$ is $4 \pi(\sinh (r / 2))^{2}$ and the length of a circle of radius $r$ is $2 \pi \sinh r$.
3. Show that any differential operator on $\mathbb{H}$ which commutes with the action of $S L_{2}(\mathbb{R})$ is a polynomial in $\Delta$ with constant coefficients.
