EXERCISE 2

1. Let ABC be a hyperbolic triangle with side lengths a = |BC|, b = |AC|, c = |AB| and angles $\alpha = \angle CAB$, $\beta = \angle ABC$, $\gamma = \angle ACB$. Show that

$$\frac{\sin \alpha}{\sinh a} = \frac{\sin \beta}{\sinh b} = \frac{\sin \gamma}{\sinh c},$$
$$\sin \alpha \sin \beta \cosh c = \cos \alpha \cos \beta + \cos \gamma,$$

and

area
$$ABC = \pi - \alpha - \beta - \gamma$$
.

2. Show that the hyperbolic area of a disc of radius r is $4\pi (\sinh(r/2))^2$ and the length of a circle of radius r is $2\pi \sinh r$.

3. Show that any differential operator on \mathbb{H} which commutes with the action of $SL_2(\mathbb{R})$ is a polynomial in Δ with constant coefficients.